Computational Mathematics

**Iterative/ Iteration method:**

In computational mathematics, an **iterative method** is a mathematical procedure that generates a sequence of improving approximate solutions for a class of problems. A specific implementation of an iterative method, including the termination criteria, is an algorithm of the iterative method. An iterative method is called **convergent** if the corresponding sequence converges for given initial approximations.

An iterative method is a powerful device of solving and finding the roots of the nonlinear equations. It is a process that uses successive approximations to obtain more accurate solutions to a linear system at each step. However, iterative methods are often useful even for linear problems involving a large number of variables (sometimes of the order if millions). It is a method involves a large number of iterations of arithmetic operations to arrive at a solution for which the computers are very often used in its process to make the task simple and efficient.

Iteration means the act of repeating a process usually with the aim of approaching a desired goal or target or result. Each repetition of the process is also called iteration and the result of the iteration are used as the starting point for the next iteration. In the problems of finding the root of an equation (or a solution of a system of equations), an iterative method uses an initial guess to generate successive approximations to a solutions.

Let the equation be f(x)=0, and the value of x to be determined.

By using the iteration method, for root finding, first we have to write equation like below:

X = f(x)

Let be an **initial approximation** of the required root, then the first approximation is given by

Similarly for second, third and so on. We can obtain the successive approximation as

Thus, we get a sequence of successive approximations which may converge to desired root.

**Example-1:**

Find the real root of the equation by iteration method.

We can rewrite the above equation by

Let

Hence the root value lies between 0 to 1

Let the initial approximation be

Since the difference between are very small, so the root is 0.75488.

**Example-2:**

Find the root of the equation: , correct to four decimal places by iteration method.

**Solution:**

Let

For x=0

For

Hence the at least one root lies between 0 and

Let the initial value (root):

The given equation may be written as:

Due to repetition of we stop our work.

So, the required root is: 0.6071 correct to four decimal places.

**Example-3:** Find the root of the equation correct to three decimal places by using iteration method.

**Solution:**

Let

For

For

So, the root lies between 1 and

The initial root

The given equation can be written as:

So,

Due to repetition of, we stop calculation. Hence, the root is: 1.523 correct to three decimal places.

**Example-4:** Find the root of the equation, using iteration method correct to four decimal places.

**Solutions:** Let,

For

For

So, the initial root lies between o and 1.

Therefore,

The equation can be written

Now,

Due to repetition of we stop calculation.

So, the required root is 0.6190 correct to four decimal places.

**Example-5:** Find the root of the equationcorrect to five decimal places by using iteration method.

Hints:

**Example-6:** Solve the equation by using iteration method.

Hints:

The root lies between 0 and 1.

**Example-7:** Solve the equationby iteration method.

Hints:

The root lies between 0 and 1. So,

**Example-8:** Starting with x=0.12, solve by using iteration method.

Hints: here,

**Example-9:** Find the root of the equation